Reynolds-Analogy Factor for a Compressible Turbulent Boundary Layer with a Pressure Gradient

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An experimental study has been made at the Naval Ordnance Laboratory for the purpose of determining the effect of a favorable pressure gradient on the Reynolds-analogy factor,§ $K = 2St/C_f$, for a natural compressible, two-dimensional turbulent boundary layer in air. Integrated values of the skin-friction coefficient and Stanton number have been determined from experimental data. The results show that Colburn's value of $K = Pr^{-2/3}$, which was found empirically from zero-pressure gradient data, is satisfactory for relating skin friction and heat transfer for the magnitudes of the pressure gradients investigated.

The turbulent boundary layer investigated was developed on the surface of a cooled stainless-steel flat plate that was placed in the diverging section of a supersonic nozzle as illustrated in Fig. 1. The Mach number distribution was determined from a continuous measurement of the ratio of Pitot to supply pressure. The resulting Mach number and Mach number gradient are plotted in Fig. 1a. The ratio of the measured wall temperatures to the supply temperature, (t_w/t_0) vs x, is plotted in Fig. 1b.

Integrated values of the skin friction were obtained from measurements of the Pitot-pressure and total-temperature profiles of the turbulent boundary layer at various stations along the plate. The total-temperature profiles were obtained using a conical equilibrium-temperature probe[1] with an apex angle of 10° and a base diameter of 0.046 in. Then the velocity and density profiles were calculated by substituting the Pitot-pressure and total-temperature profile measurements into the perfect-gas relationships and assuming that $p(y) = \text{const}$. By numerical integration, it then was possible to calculate the momentum and displacement thicknesses. Figure 1c contains a plot of these thicknesses along with some values calculated from the theory of Ref. 2. The calculations were made using the experimental δ^* and θ at $x = 1.28$ ft as initial values. The theoretical predictions are in good agreement with the experimental data. Now these thicknesses along with the experimentally determined Mach number distribution can be placed in the following relationship for the integrated skin-friction coefficient:

$$\int_{x_1}^{x_n} \frac{C_f}{2}\, dx = \int_{x_1}^{x_n} d\theta + \int_{x_1}^{x_n} \frac{\theta}{M_e} \frac{[2 + (\delta^*/\theta) - M_e^2]}{\{1 + [(\gamma - 1)/2]M_e^2\}}\, dM_e$$

which is the momentum-integral equation integrated with respect to x. Again the integration was performed numerically starting at $x_1 = 1.28$ ft. Plotted on Fig. 2 are both the experimental (shown as circles) and theoretical (solid line) values of

$$\int_{x_1}^{x_n} \frac{c_f}{2}\, dx$$

vs x. The latter were calculated using the skin-friction law of Ref. 2 and agree quite well with experiment.

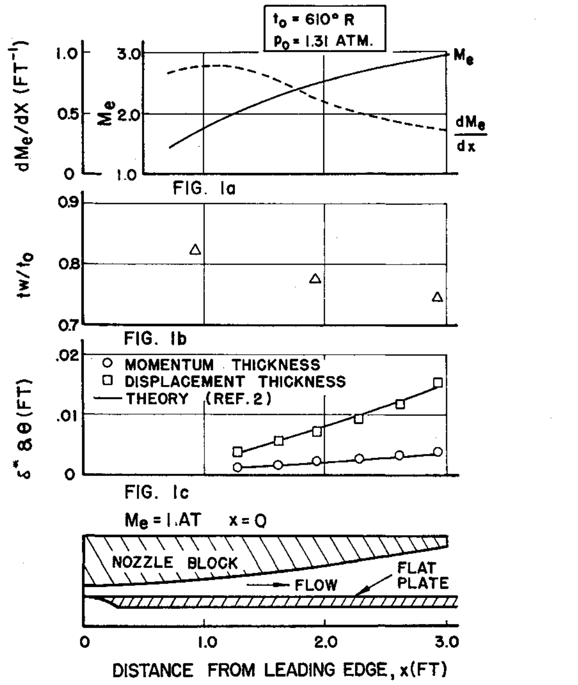

Fig. 1 Mach number and Mach number gradient, momentum and displacement thickness, and wall temperatures along the flat plate

The Stanton numbers also were determined from experimental data using the following relationship:

$$St = K_{ss} \left(\frac{\partial t}{\partial y}\right)_w \frac{1}{(h_{aw} - h_w)\rho_e u_e}$$

where the temperature gradient in the plate evaluated at the wall, $(\partial t/\partial y)_w$, was obtained from the measured steady-state temperature distributions in the flat plate, taking into consideration the variation of the thermal conductivity of the stainless steel (K_{ss}) with temperature.

If the proper value for K is chosen, then

$$\int_{x_1}^{x_n} \frac{C_f}{2}\, dx = \int_{x_1}^{x_n} \frac{St}{K}\, dx$$

Reference 2 suggests using the following relationship:

$$K = Pr^{-2/3}\exp\{1.561[(\delta^*/\theta)_i - (\delta^*/\theta)_{i,fp}]\}$$

which can be written as

$$K = Pr^{-2/3}\exp\left\{1.561\left[\frac{t_e}{t_0}\left(\frac{\delta^*}{\theta} - \frac{\gamma - 1}{2} M_e^2\right) - 1.3\,\frac{t_w}{t_0}\right]\right\}$$

when $(\delta^*/\theta)_{i,fp} = 1.3$, and $C_p = \text{const}$. Experimental data

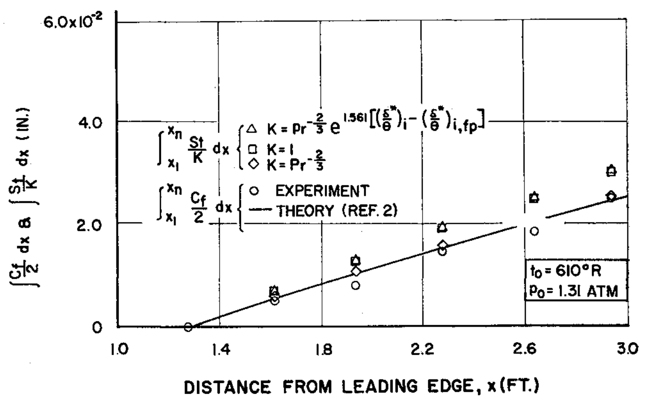

Fig. 2 Comparison between $\int (C_f/2)dx$ and $\int (St/K)dx$ for various values of K

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§ The nomenclature of Ref. 2 is used unless defined.

were used for calculating K. With this value of K, the values of

$$\int_{x_1}^{x_n} \frac{St}{K}\,dx$$

are somewhat higher than the values of

$$\int_{x_1}^{x_n} \frac{C_f}{2}\,dx$$

as shown in Fig. 2. It is of interest to note that, when $K = 1$, then the values of

$$\int_{x_1}^{x_n} \frac{St}{K}\,dx$$

are nearly the same as the values of Ref. 2. An empirical value for K for the zero-pressure gradient case has been found by Colburn,[3] which is simply $K = Pr^{-2/3}$. Using Colburn's K, the values of

$$\int_{x_1}^{x_n} \frac{St}{K}\,dx$$

obtained are in good agreement with the integrated values of the skin-friction coefficient.

References

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[2] Reshotko, E. and Tucker, M., "Approximate calculation of the compressible turbulent boundary layer with heat transfer and arbitrary pressure gradient," NACA TN 4154 (December 1957).

[3] Colburn, A. P., "A method of correlating forced convection heat-transfer data and a comparison with fluid friction," Trans. Am. Inst. Chem. Engrs. 29, 174–210 (1933).

Radiation Slip

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It long has been recognized (see, e.g., Goulard[1]) that a close analogy exists between radiative transfer processes, as ascribed to the motion of a photon gas, and molecular transport phenomena. In analyzing radiative heat transfer between a gas and a solid surface, it soon is apparent that the analogy to low-density conductive heat transfer is sufficiently close that many of the concepts should be interchangeable. On this basis, we propose that the difficult problem of calculating radiative heat transfer between a gas and a surface in the "transition" regime, between an optically thick (opaque) medium and an optically thin (transparent) one, may be circumvented by the use of a simple rarefied flow model. The model uses the radiation conduction equation, valid for an optically dense medium (Rosseland or diffusion approximation[2]), in conjunction with what we shall term a "radiation slip" boundary condition.

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The radiation slip condition is analogous to the Maxwell velocity slip in a low-density molecular gas, where there is a discontinuity at the surface between the gas and wall velocity. It is, however, more specifically analogous to the Smoluchowski temperature jump, in which a discontinuity between the gas and wall temperature exists in a rarefied molecular medium. The proposed boundary condition may be stated: in an absorbing and emitting radiating medium there is, adjacent to a solid surface, a temperature discontinuity that is proportional to the local photon mean free path multiplied by the radiation temperature gradient.

To illustrate the present idea, the calculation of the radiative heat transfer for an absorbing-emitting gas contained between infinite parallel walls separated by a distance Δ and maintained at different uniform temperatures is considered (see Fig. 1). In order to bring out the main features of the present approach, at first the gas will be treated as nonconducting, in the sense that it does not transfer heat by molecular conduction. Furthermore, both walls are taken to be blackbodies so that they are perfect absorbers of any incident radiation. The problem is to determine, say, for a fixed spacing between the plates, how the net radiative heat transfer rate to the cooler wall changes as the photon mean free path in the gas is varied. The photon mean free path l_ν is defined here as the inverse of the volumetric absorption coefficient frequently used in radiative engineering calculations. The nature of the radiative transfer then will be characterized by the appropriate dimensionless optical length τ, which, for the present problem, is

$$\tau = \Delta/l_\nu \qquad (1)$$

This parameter is analogous to the inverse Knudsen number of low-density fluid mechanics, where it is recalled that the Knudsen number is the ratio of the molecular collision mean free path to the appropriate characteristic flow length.

Two limits can be distinguished immediately. In one limit, the density is so low that the photon mean free path (for all wavelengths) is large enough that the gas may be considered optically thin ($\tau \ll 1$). Under this condition, the net radiative heat transfer rate between the walls is determined simply by the blackbody flux from the walls and is given by

$$-q = \sigma(T_2^4 - T_1^4) \qquad \tau \ll 1 \qquad (2)$$

Here T_2 is the absolute temperature of the hot plate, T_1 that of the cold plate, and σ is the Stefan-Boltzmann constant.

In the opposite limit of an optically thick gas ($\tau \gg 1$), the density is so large that the photons are trapped in the gas. The photon mean free path is then very small, and radiation emitted at a point is absorbed at a distance comparable to

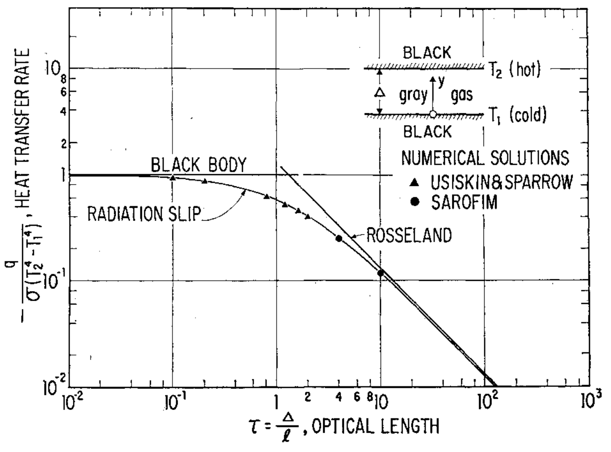

Fig. 1 Radiative heat transfer rate between two infinite isothermal parallel black plates containing a gray gas in the gap between the plates